In: *Representation and Reality in Humans, Other Living Organisms and Intelligent Machines*, Dodig-Crnkovic, Gordana and Giovagnoli, Raffaela (Eds.), Springer International Publishing, Berlin-New York, 2017, pp. 177-210

The quantum field theory (QFT) dual paradigm in fundamental physics and the semantic information content and measure in cognitive sciences

Gianfranco Basti[1]

Abstract. In this paper we explore the possibility of giving a justification of the "semantic information" content and measure, in the framework of the recent coalgebraic approach to quantum systems and quantum computation, extended to QFT system. In QFT, indeed, any quantum system has to be considered, as an "open" system, because always interacting with the background fluctuations of the quantum vacuum. Namely, the Hamiltonian in QFT is always including the quantum system and its inseparable thermal bath, formally "entangled" like an algebra with its co-algebra, according to the principle of the "doubling" of the degrees of freedom (DDF) between them. This is the core of the representation theory of the cognitive neuroscience based on QFT. Moreover, in QFT the probabilities of the quantum states follow a Wigner distribution, based on the notion and the measure of quasi-probability where regions integrated under given expectation values do not represent mutually exclusive states. This means that a computing agent, either natural or artificial in QFT, against the Quantum Turing Machine paradigm, is able to change dynamically the representation space of its computations. This depends on the possibility of interpreting the QFT system computations within the framework of the Category Theory logic and its principle of duality between opposed categories, such as the algebra and coalgebra categories of the QFT. This allows us to justify and not only to suppose, like in the "Theory of Strong Semantic Information" of L. Floridi, the definition of modal "local truth" and the notion of semantic information as a measure of it, despite both measures are defined on quasi-probability distributions.

Keywords: semantic information, cognitive neuroscience, quantum field theory, quantum mechanics, quantum computations, coalgebraic modal logic, local truth, category theory.

1 Introduction: a paradigm shift

Perhaps, the better synthesis of the actual paradigm shift in fundamental physics is the positive answer that it seems necessary to give to the following question: "Is physics legislated by cosmogony?". Such a question is the title of a visionary paper wrote in 1975 by J. A. Wheeler and C. M. Patton and published in the first volume of a fortunate series of the Oxford University about the quantum gravity [1].

Such a revolution, suggesting a dynamic justification of the physical laws, originally amounts to the so-called *information theoretic approach* in quantum physics as the natural science counterpart of a *dual ontology* taking information and energy as two fundamental magnitudes in basic physics and cosmology. This approach started from Richard Feynman's influential speculation that a quantum computer could simulate any physical system [2]. This is the meaning of the famous posit "it from bit" principle stated by R. Feynman's teacher, J. A. Wheeler [3, p. 75]. The cornerstones of this reinterpretation are, moreover, D. Deutsch's demonstration of the universality of the Quantum Universal Turing Machine (QTM) [4], and overall C. Rovelli's development of a *relational* Quantum Mechanics QM [5]. An updated survey of such an informational approach to fundamental physics is in the recent collective book, edited by H. Zenil, and with contributions, among the others, of R. Penrose, C. Hewitt, G. J. Chaitin, F. A. Doria, E. Fredkin, M. Hutter, S. Wolfram, S. Lloyd, besides the same D. Deutsch [6].

There are, however, several theoretical versions of the information theoretic approach to quantum physics. It is not important to discuss all of them here (for an updated list in QM, see, for instance [7]), even though all can be reduced to essentially two.

[1] Faculty of Philosophy, Pontifical Lateran University, Rome. basti@pul.it

1. The first one is the classical “infinitistic” approach to the *mathematical physics* of information in QM. Typical of this approach is the notion of the *unitary evolution* of the *wave function*, with the connected, supposed *infinite* amount of information it “contains”, “made available” in different spatio-temporal cells via the mechanism of the “decoherence” of the wave function. Finally, essential for this approach is the necessity of supposing *an external observer* (“information for whom?” [7]) for the foundation of the notion and of the measure of information. This is ultimately Shannon’s purely syntactic, measure and notion of information in QM [5]. Among the most prominent representatives of such an approach, we can quote the German physicist H. D. Zeh [8, 9] and the Swedish physicist, M. Tegmark [10].
2. The second approach, the emergent one today, is related to a “finitary”[2] approach to the *physical mathematics* of information, taken as a fundamental physical magnitude together with energy. It is related to Quantum Field Theory (QFT), because of the possibility it gives of spanning the microphysical, macrophysical, and even the cosmological realms, within one only quantum theoretical framework, differently from QM [11].

In this contribute we discuss the relevance of the second approach for the theory of the *semantic information*, both in biological and cognitive sciences.

2 From QM to QFT in fundamental physics

The notion of quantum vacuum is fundamental in QFT. This notion is the only possible explanation at the fundamental microscopic level, of the *third principle of thermodynamics* (“The entropy of a system approaches a constant value as the temperature approaches zero”). Indeed, the Nobel Laureate Walter Nernst, first discovered that for a given mole of matter (namely an ensemble of an Avogadro number of atoms or molecules), for temperatures close to the absolute 0, T_0, the variation of entropy ΔS would become infinite (by dividing by 0).

Nernst demonstrated that for avoiding this catastrophe we have to suppose that the molar heat capacity C is not constant at all, but vanishes, in the limit $T \rightarrow 0$, so to make ΔS finite, as it has to be. This means however, that near the absolute 0°C, there is a mismatch between the variation of the body content of energy, and the supply of energy from the outside. We can avoid such a paradox, only by supposing that such a mysterious inner supplier of energy is the vacuum. This implies that the absolute 0°C is unreachable. In other terms, there is an unavoidable fluctuation of the elementary constituents of matter. The ontological conclusion for fundamental physics is that we cannot any longer conceives physical bodies as isolated.

> The vacuum becomes a bridge that connects all objects among them. No isolated body can exist, and the fundamental physical actor is no longer the atom, but the field, namely the atom space distributions variable with time. Atoms become the “quanta” of this matter field, in the same way as the photons are the quanta of the electromagnetic field [12, p. 1876].

For this discovery, eliminating once forever the notion of the “inert isolated bodies” of the Newtonian mechanics, Walter Nernst is a chemist who is one of the founders of the modern quantum physics.

Therefore, the theoretical, core difference between QM and QFT can be essentially reduced to the criticism of the classical interpretation of the QFT as a “second quantization” a to the QM. In QFT, indeed, the classical Stone-Von Neumann theorem [13] does not hold. This theorem states that, for system with a *finite* number of degrees of freedom, which is always the case in QM, the representations of the canonical commutation relations (CCR’s)[3] are all *unitarily equivalent to each other*, so to justify the exclusive use of Shannon information in QM.

---

[2] For the notion of “finitary” computation, as distinguished from “infinitistic” (second-order computation) and “finististic” (Turing-like computation) see [75]. This notion depends on the Category Theory (CT) interpretation of logic and computation [56], as far as based on Aczel’s NWF set theory [64], justifying a *coalgebraic semantics* in quantum computing [78], as far as based on the CT principle of the *dual equivalence* between a Boolean initial algebra and a final coalgebra [66, 67]. The key notion of the *doubling of the degrees of freedom* between a *q*-deformed Hopf algebra and a *q*-deformed Hopf coalgebra, as representing each quantum system in quantum field theory, satisfies perfectly such a logic, as we see below.

[3] It is useful to recall here that the *canonical variables* (e.g., position and momentum) of a quantum particle do not commute among themselves, like in classical mechanics, because of Heisenberg’s uncertainty principle. The fundamental discovery of D. Hilbert consists in demonstrating that each canonical variable of a quantum particle commutes with the Fourier transform of the other (such a relationship constitutes a CCR), so to allow a geometrical representation of all the states of a quantum system in terms of a commuting variety, i.e., the relative “Hilbert space”.

On the contrary, in QFT systems, the number of the degrees of freedom is not finite, so that infinitely many unitarily inequivalent representations of the canonical commutation (bosons) and anti-commutation (fermions) relations exist. Indeed, through the principle of the *Spontaneous Symmetry Breaking* (SSB) in the vacuum ground state, infinitely (not denumerable) many, quantum vacuum conditions, compatible with the ground state, there exist. Moreover, this holds not only in the relativistic (microscopic) domain, but also it applies to non-relativistic many-body systems in condensed matter physics, i.e., in the macroscopic domain, and even on the cosmological scale [11, pp. 18. 53-96].

Indeed, starting from the discovery, during the 60's of the last century, of the dynamically generated long-range correlations mediated by the *Nambu-Goldstone bosons* (NGB) [14, 15], and hence for their role in the local gauge theory by the Higgs field, the discovery of these collective modes changed deeply the fundamental physics. Before all, it appears as an effective, alternative method to the classically Newtonian paradigm of the perturbation theory, and hence to its postulate of the asymptotic condition.

In this sense, "QFT can be recognized as an *intrinsically thermal* quantum theory" [11, p. ix]. Of course, because of the intrinsic character of the thermal bath, the whole QFT system can recover the classical Hamiltonian character, because of the necessity of anyway satisfying the energy balance condition of each QFT (sub-)system with its thermal bath ($\Delta E = 0$), mathematically formalized by the "algebra doubling", between a *q*-deformed Hopf algebra and its "dual" (see note 2) *q*-deformed Hopf co-algebra, where *q* is a thermal parameter [16].

Therefore, in QFT an uncertainty relation holds, similar to the one of Heisenberg, relating the uncertainty on the number of the field quanta to the one of the field phase, namely:

$$\Delta n \Delta \varphi \geq \varphi(\hbar)$$

Where *n* is the number of quanta of the force field, and φ is the field phase. If ($\Delta n = 0$), φ is undefined so that it makes sense to neglect the waveform aspect in favor of the individual, particle-like behavior. On the contrary if ($\Delta\varphi = 0$), *n* is undefined because an extremely high number of quanta are oscillating together according to a well-defined phase, i.e., within a given phase coherence domain. In this way, it would be nonsensical to describe the phenomenon in terms of individual particle behavior, since the collective modes of the force field prevail.

In QFT there is duality between *two dynamic entities*: the fundamental force field and the associated quantum particles that are simply the quanta of the associated field that is different for different types of particles. In such a way, the quantum entanglement does not imply any odd relationship between particles like in QM, but simply it is an expression of the unitary character of a force field. To sum up, according to such more coherent view, Schrödinger wave function of QM appears to be only a statistical coverage of a finest structure of the dynamic nature of reality.

3 QFT of dissipative structures in biological systems

3.1 Order and vacuum symmetry breakdown

It is well-known that a domain of successful application of QFT is the study of the microphysics of condensed matter, that is in systems displaying at the macroscopic level a high degree of coherence related to an *order parameter.* The "order parameter", that is the macroscopic variable characterizing the new emerging level of matter organization, is related to the *matter density distribution*. In fact, in a crystal, the atoms (or the molecules) are "ordered" in well-defined positions, according to a *periodicity law* individuating the crystal lattice.

Other examples of such ordered systems in condensed matter realm are the magnets, the lasers, the super-conductors, etc. In all these systems the emerging properties related to the respective order parameters, are neither the properties of the elementary constituents, nor their "summation", but new properties depending on *the modes in which they are organized*, and hence on *the dynamics controlling their interactions.* In this way, at each new macroscopic structure, such a crystal, a magnet or a laser, corresponds a new "function", the "crystal function", the "magnet function", etc.

Moreover, all these emerging structures and functions are controlled by *dynamic parameters,* that, with an engineering terminology, we can define as *control parameters.* Changing one of them, the elements can be subject to different dynamics with different collective properties, and hence with different collective behaviors and functions. Generally, the temperature is the most important of them. For instance, crystals beyond a given critical temperature — that is different for the different materials — lose their crystal-like ordering, and the elements acquire as a whole the macroscopic structure-functions of an amorphous solid or, for higher temperatures, they lose any static structure, acquiring the behavior-function of a gas.

So, any process of *dynamic ordering,* and of *information gain,* is related with a process of *symmetry breakdown.* In the magnet case, the "broken symmetry" is the rotational symmetry of the magnetic dipole of the electrons, and the "magnetization" consists in the correlation among all (most) electrons, so that they all "choose", among all the directions, that one proper of the magnetization vector.

To sum up, whichever dynamic ordering among many objects implies an "order relation", i.e., a *correlation* among them. What, in QFT, at the *mesoscopic/macroscopic* level is denoted as *correlation waves* among molecular structures and their chemical interactions, at the *microscopic* level any correlation, and more generally any interaction, is mediated by *quantum correlation particles.* They are called "Goldstone bosons" or "Nambu-Goldstone Bosons (NGB)" [17, 14, 15], with mass — even though always very small (if the symmetry is not perfect in finite spaces) —, or *without mass at all* (if symmetry is perfect, in the abstract infinite space). Less is the inertia (mass) of the correlation quantum, greater is the distance on which it can propagate, and hence the distance on which the correlation (and the ordering relation) constitutes itself.

However, an important *caveat* is necessary to do about the different role of the "Goldstone bosons" as quantum correlation particles, and the "bosons" of the different energy fields of quantum physics (QED and QCD). These latter are the so-called *gauge bosons*: the photons γ of the electromagnetic field; the gluons g of the strong field, the bosons $W^{\pm}$ and the boson Z of the electroweak field; and the scalar Higgs boson H^0 of the Higgs field, common to all the precedent interactions.

The gauge bosons are properly mediators of the *energy exchanges,* among the interacting elements they correlate, because they are effectively quanta of the energy field they mediate (e.g., the photon is the quantum of the electromagnetic field). Therefore, the energy quanta are bosons able to change the *energy state* of the system. For instance, in QED of atomic structures, they are able to change the fundamental state (minimum energy), into one of the excited states of the electronic "cloud" around the nucleus.

On the contrary, NGB correlating quanta are not mediators of the interactions among the elements of the system. They determine only the *modes of interaction* among them. Hence, any symmetry breakdown in the QFT of condensed matter of chemical and biological systems has one only gauge boson mediator of the underlying energy exchanges, the photon, since they all are electromagnetic phenomena. Therefore, the phenomena here concerned, from which the emergence of *macroscopic* coherent states derives, implies the generation, effectively the *condensation*, of correlation quanta with negligible mass, in principle null: the NGB, indeed. This is the basis of the fundamental "Goldstone theorem" [18, 19]. The NGB bosons acquire different names for the different modes of interaction, and hence of the coherent states of matter they determine – *phonons* in crystals, *magnons* in magnets, *polarons* in biological matter, etc.. Indeed, what characterizes the coherent domains in living matter is the phase coherence of the *electric dipoles* of the organic molecules and of the water, in which only the biomolecules are *active.* Therefore, despite the correlation quanta are real particles, observable with the same techniques (diffusion, scattering, etc.), not only in QFT of condensed matter, but also in QED and in QCD like the other quantum particles, wherever we have to deal with broken symmetries [15], nevertheless they do not exist *outside* the system they are correlating. For instance, without a crystal structure (e.g., by heating a diamond over 3,545 °C), we have still the component atoms, but no longer phonons. Also and overall in this aspect, the correlation quanta differ from energy quanta, like photons. Because the gauge bosons are *energy* quanta, they cannot be "created and annihilated" without residuals.

Better, in any quantum process of particle "creation/annihilation" in quantum physics, what is conserved is the energy/matter, mediated by the energy quanta (gauge bosons), not their "form", mediated by the NGB correlation quanta. Also on this regard, a dual ontology (matter/form) is fundamental for avoiding confusions and misinterpretations in quantum physics.

Moreover, because the mass of the correlation quanta is in any case negligible (or even null), *their condensation does not imply a change of the energy state of the system.* This is the fundamental property for understanding how, not only the stability of a crystal structure, but also the relative stability of the living matter structures/functions, at different levels of its self-organization (cytoskeleton, cell, tissue, organ…), can depend on such basic *dynamic* principles. In fact, all this means that, if the symmetric state is a fundamental state (a minimum of the energy function corresponding to a *quantum vacuum* in QFT of dissipative systems), also the ordered state, after the symmetry breakdown and the instauration of the ordered state, remains *a state of minimum energy*, so to be *stable* in time. In kinematics terms, it is a *stable attractor* of the dynamics.

3.2 The Doubling of Degrees of Freedom (DDF) in QFT and in neuroscience

We said that the relevant quantum variables in biological systems are the *electrical dipole vibrational modes* in the water molecules, constituting the oscillatory "dynamic matrix" in which also neurons, glia cells, and the other mesoscopic units of the brain dynamics are dipped. The condensation of massless NGB (polarons) — corresponding, at the mesoscopic level, to the long-range correlation waves observed in brain dynamics — depends on the triggering action of the external stimulus for the symmetry breakdown of the quantum vacuum of the corresponding brain state. In such a case, the "memory state" corresponds to a coherent state for the basic quantum variables, whose mesoscopic order parameter displays itself as the amplitude and phase modulation of the carrier signal.

In the classical Umezawa's model of brain dynamics [20], however, the system suffered in an "intrinsic limit of memory capacity". Namely, each new stimulus produces the associated polaron condensation, by cancelling the precedent one, for a sort of "overprinting". *This limit does not occur in dissipative QFT where the many-body model predicts the coexistence of physically distinct patterns, amplitude modulated and phase modulated*. That is, by considering the brain as it is, namely an "open", "dissipative" system continuously interacting with its environment, there not exists one only ground (quantum vacuum) state, like in the thermal field theory of Umezawa, where the system is studied at equilibrium. On the contrary, in principle, there exists infinitely many ground states (quantum vacuum's), so to give the system a potentially infinite capacity of memory. To sum up, the solution of the overprinting problem relies on three facts [21]:

1. In a dissipative (non-equilibrium) quantum system, there are (in principle) infinitely many quantum vacuum's (ground or zero-energy) states, on each of which a whole set of non-zero energy states (or "state space" or "representation states") can be built.
2. Each input triggers one possible irreversible time-evolution of the system, by inducing a "symmetry breakdown" in one quantum vacuum, i.e., by inducing in it an ordered state, a coherent behavior, effectively "freezing" some possible degrees of freedom of the constituting elements behaviors (e.g., by "constraining" them to oscillate on a given frequency). At the same time, the input "labels" dynamically the induced coherent state, as an "unitary non-equivalent state" of the system dynamics. In fact, such a coherent state persists in time as a ground state (polarons are not energetic bosons, are Goldstone bosons) so to constitute a specific "long-term" memory state for such a specific coupling between the brain dynamics and its environment. On the other hand, a brain that is no longer dynamically coupled with its environment is, either in a pathological state (schizophrenia), or it is simply dead.
3. At this point emerges the DDF principle as a both physical and mathematical necessity of such a brain model. Physical, because a dissipative system, even though in non-equilibrium, must anyway satisfy the *energy balance*. Mathematical, because the 0 energy balance requires a "doubling of the system degrees of freedom". The *doubled* degrees of freedom, say $\tilde{A}$ (the tilde quanta, where the non-tilde quanta *A* denote the brain degrees of freedom), thus represent the environment to which the brain state is coupled. The environment (state) is thus represented as the "time-reversed *double*" of the brain (state) on which it is impinging. The environment is hence "modeled on the brain", but according to the finite set of degrees of freedom *the environment itself elicited* in the brain.

What is relevant for our aims, is that to each set of degrees of freedom *A* and to its "entangled doubled" $\tilde{A}$ is relative a *unique number* $\mathcal{N}$, i.e. $\mathcal{N}_A, \mathcal{N}_{\tilde{A}}$ that in module, $|\mathcal{N}|$, *identifies univocally*, i.e., it *dynamically labels*, a given *phase coherence domain*, i.e., a quantum system state entangled with its thermal bath state, in our case, *a brain state matching its environment state.* This depends on the fact that generally, in the QFT mathematical formalism the number $\mathcal{N}$ is a numeric value expressing the NGB condensate value from which a phase coherence domain *directly depends*. In an appropriate *set theoretic interpretation,* because for each "phase coherence domain" *x,* effectively $|\mathcal{N}|$ *identifies univocally* such a domain, it corresponds to an "identity function Id_x" that, in a "finitary" coalgebraic logical calculus, corresponds to the *predicate satisfied by such a domain because identifying univocally it.* In other terms, Vitiello's reference to the predicate "magnet function" or "crystal function" we quoted at the beginning of sect. 3.1 are not metaphors, but are expressions of a fundamental formal tool – the "co-membership notion" – of the coalgebraic predicate calculus (see below sect. 5.2). Therefore, of the DDF applied to the quantum foundation of the cognitive neuroscience we have illustrated elsewhere its logical relevance, for an original solution of the reference problem (see [22, 23]).

There exists a huge amount of experimental evidence in brain dynamics of such phenomena, collected by W. Freeman and his collaborators. This evidence found, during the last ten years, its proper mathematical modeling in the dissipative QFT approach of Vitiello and his collaborators, so to justify the publication during the last years of several joint papers on these topics (see, for a synthesis, [24, 25]).

To sum up [26], Freeman and his group used several advanced brain imaging techniques such as multi-electrode EEG, electro-corticograms (ECoG), and magneto-encephalogram (MEG) for studying what neurophysiologist generally consider as the *background activity* of the brain, often filtering it as "noise" with respect to the synaptic activity of neurons they are exclusively interested in. By studying these data with computational tools of signal analysis to which physicists, differently from neurophysiologists, are acquainted, they discovered the massive presence of patterns of AM/FM phase-locked oscillations. They are intermittently present in resting and/or awake subjects, as well as in the same subject actively engaged in cognitive tasks requiring interaction with the environment. In this way, we can describe them as features of the background activity of brains, modulated in amplitude and/or in frequency by the "active engagement" of a brain with its surround. These "wave packets" extend over coherence domains covering much of the hemisphere in rabbits and cats [27, 28, 29, 30], and regions of linear size of about 19 cm in human cortex [31], with near zero phase-dispersion [32]. Synchronized oscillations of large-scale neuron arrays in the β and γ ranges are observed by MEG imaging in the resting state and in the motor-task related states of the human brain [33].

4 Semantic information in living and cognitive systems

4.1 QFT systems and the notion of negentropy

Generally, the notion of information in biological systems is a synonym of the *negentropy* notion, according to E. Schrödinger's early use of such a term. Applied, however, to QFT foundations of dissipative structures in biological systems, the notion of negentropy is not only associated with the *free energy*, as Schrödinger himself suggested [34], but also with the notion of *organization,* as the use of this term by A. Szent-György first suggested [35]. The notion of negentropy it is thus related with the constitution of *coherent domains* at different space-time scales, as the application of QFT to the study of dissipative structures demonstrates, since the pioneering H. Frölich works [36, 37].

On this regard, it is important to emphasize also the key-role of the notion of *stored energy* that such a multi-level spatial-temporal *organization* in coherent domains and sub-domains implies (i.e., the notion of quantum vacuum "foliation" in QFT), as distinct from the notion of *free energy* of classical thermodynamics [38]. Namely, as we know from the precedent discussion, the constitution of coherent domains allows chemical reactions to occur at *different time-scales,* with a consequent energy release, that so becomes immediately available exactly *where/when it is necessary*. For instance, resonant energy transfer among molecules occurs typically in 10^{-14} sec., whereas the molecular vibrations themselves die down, or thermalize, in a time between 10^{-9} and 10^{1}sec. Hence, it is a 100% highly efficient and highly specific process, being determined by the frequency of the vibration itself, given that resonating molecules can attract one another. Hence, the notion of "stored energy" is meaningful at every level of the complex spatial-temporal structure of a living body, from the single molecule to the whole organism.

This completes the classical thermodynamic picture of L. Szilard [39] and L. Brillouin [40] according to which the "Maxwell demon", for getting information so to compensate the entropic decay of the living body must consume free energy from the environment. This means an increasing of the global entropy according to the dictate of the Second Law. However, this has to be completed in QFT with the evidence coming from the Third Law discussed in this paper.

This occurs at the maximum level in the biological realm in the human brain dynamics. For illustrating this point as to the DDF applied in neuroscience, Freeman and his collaborators spoke about "dark energy" as to the extreme reservoir of energy hidden in human brain dynamics. Human brain indeed has 2% of the human body mass, but dissipates 20-25% of the body resting energy. This depends on the extreme density of cells in the cortices ($10^5/mm^3$), with an average of 10^4 connections [41].

To conclude this discussion we showed that the "dual paradigm" related to the QFT interpretation of the "information theoretic" approach to quantum physics is not depending on the distinction between "energy" and "information", like in the QM interpretation, where the "information" notion and measure – differently from the "energy" ones – are "observer-related", and therefore, logically, is only "syntactic". In the QFT interpretation where "information" is a physical magnitude, i.e., it is a thermodynamic *negentropy,* the duality concerns the two components of the negentropy notion and measure. They are, respectively, the *energetic* component (quantum "gauge bosons") and the *ordering* component (quantum “Nambu-Goldstone bosons”) of a phase coherence domain, including the two entangled quantum states of the system and of its environment.

On the other hand, precisely because “ordering” is also a fundamental semantic notion in set-theoretic logic, the “semantic information” notion and measure are strictly depending on the *logical* and *mathematical* notion of “duality”. This duality in category theory logic concerns two opposed categories, specifically, in theoretical computer science (TCS), the notion of the “dual equivalence” between an algebra and its coalgebra, on which the notion of “local truth” and “finitary computation”, on one hand, as well as the notion and measure of *semantic information,* on the other one, strictly depend.

These two notions of “duality”, physical and logical, are however strictly interconnected in QFT, because both depend on the notion of *NGB condensates*, as constituting, respectively, the “ordering” component of the negentropy in information physics, and the sufficient condition for interpreting the QFT systems as computing systems. A short introduction to all these notions will be the object of the rest of this paper, in the framework of the recent “coalgebraic approach” to quantum computing in TCS.

4.2 Syntactic vs. semantic information in quantum physics

4.2.1 Shannon’s syntactic theory of information in QM and in the mathematical communication theory

It has been emphasized the Shannon nature of the notion and measurement of information that can be associated to the decoherence in QM, overall in the relational and hence computational interpretations of QM illustrated above [5]. In fact, in both cases the “information” can be associated to the uncertainty H removal, in the sense that “more probable” or “less uncertain” an event/symbol is, less informative (or, psychologically, less “surprising”) its occurrence is. Mathematically, in the Mathematical Theory of Communication (MTC), the information H associated with the i^{th} symbol x among N ones (= alphabet), can be defined as:

$$H = \sum_{i=1}^{N} p(x_i) I(x_i) = -\sum_{i=1}^{N} p(x_i) \log p(x_i)$$

Where $p(x_i)$ is the relative probability of the i^{th} symbol x as to the N possible ones, I is the information content associated with the symbol occurrence, that is the inverse of its relative probability (less probable it is, more informative its occurrence is). The information amount H has thus the dimensions of a statistical *entropy* that is very close to the thermodynamic entropy S of statistical mechanics:

$$S = -k_B \sum_i p(x_i) \log p(x_i)$$

Where x_i are the possible microscopic configurations of the individual atoms and molecules of the system (microstates) which could give rise to the observed macroscopic state (macrostate) of the system, and k_B is the Boltzmann constant. Based on the correspondence principle, S is equivalent in the classical limit, i.e. whenever the classical notion of probability applies, to the QM definition of entropy by John Von Neumann:

$$S = -k_B \mathrm{Tr}(\rho \log \rho)$$

Where ρ is a density matrix and $\langle \mathrm{Tr} \rangle$ is the trace operator of the matrix. Indeed, who suggested Claude Shannon to denote as “entropy” the statistical measure of information H he discovered, was the same Von Neumann. The informativeness or the “uncertainty (removal)” associated with (the occurrence of) a symbol in MTC (or with an event in statistical classical and quantum mechanics) is (are) only “syntactic” and not “semantic” [42, p. 3]. Effectively, the symbol (the event) occurs as *uninterpreted* (context-independent) and *well-formed* (determined), according to the *rules* of a *fixed* alphabet or code (i.e., according to the *unchanged laws* of physics.

Anyway, starting from the pioneering works of D. M. Mackay [43], and of R. Carnap & Y. Bar-Hillel [44], it is a *leitmotiv,* in almost any work dealing with the notion of information in biological and cognitive systems, the vindication of the *semantic/pragmatic* character of it. Particularly, because information concerns here self-organizing and complex processes, in them the “evolution of coding”, and the notion of “local (contingent) truth” (semantics), in the sense of *ad-*

equacy for an optimal fitting with the environment (pragmatics), are essential [45, 46, 47]. More specifically, in QFT differently from QM, it is significant the pragmatic information content, defined as the ratio of the rate of energy dissipation (power) to the rate of decrease in entropy (negentropy) [47]. A measure generally considered in literature as the proper information measure of self-organizing systems. Evidently, in the DDF formalism of QFT, the relationship between a quantum system and its thermal bath (environment), and specifically, in neuroscience, the relationship between the brain and its contextual environment, the notion and measure of pragmatic information, as described in [47], plays an essential role [41].

What is here to be emphasized, before all, is that in QFT the Wigner function (WF), on which the probabilities of the physical states are calculated, is deeply different from the Schrödinger wave function of QM, not only because the former, differently from the latter, is defined on the phase space of the system. What is much more fundamental is that the WF uses the notion of *quasi-probability* [48], and not the notion of probability of the classical Kolmogorov axiomatic theory of probability [49].

Indeed, the notion of quasi-probability allows regions integrated under given expectation values do not represent *mutually exclusive states*, so to violate one of the fundamental axioms of Kolmogorov's theory. I.e., the separation of variables in such distributions is not fixed, but, as it is the rule in the case of phase transitions, can evolve dynamically (see the QFT interpretation of the "quantum uncertainty principle" at the end of sect. 2). From the computability theory standpoint, this means that a physical system in QFT, against the TM and QTM paradigms, is able to change dynamically "the basic symbols" of its computations, since new collective behaviors can emerge from individual ones, or vice versa. In this way, this justifies the definition of the information associated with a WF as a "semantic information content".

The semantic information in QFT computations hence satisfies, from the logical standpoint, the notion of *contingent*, or better, *local truth* so to escape the Carnap & Bar-Hillel paradoxes (CBP) [44]. To introduce this notion, it might be "pedagogically" useful to discuss briefly the "Theory of Strong Semantic Information" (TSSI) developed by L. Floridi, essentially because it shares with QFT the same notion of quasi-probability. Even though, in the QFT usage of the "quasi-probability" notion there is no necessity of violating also the other axiom of Kolmogorov's axiomatic theory of probability, i.e., the axiom excluding the "negative probabilities" that, on the contrary, Floridi uses in its – also for this reason – only "pedagogical" approach to the notion of "local/contingent truth" [50].

4.2.2 Floridi's semantic information thory

Following the critical reconstruction of both the theories (CSI and TSSI), by S. Sequoiah-Grayson [51], CSI approach is based on Carnap's theory of intensional modal logic [52]. In this theory, given n individuals and m monadic predicates, we have 2^{nm} possible worlds and 2^m Q-predicators, intended as individuations of possible type of objects, given a conjunction of primitive predicates either un-negated or negated. A full sentence of a Q-predicator is a Q-sentence, hence a possible world is a conjunction of n Q-sentences, as each Q-sentence describes a possible existing individual. The *intension* of a given sentence is taken to be the set of possible worlds that make true the sentence, i.e., included by the sentence. This is in relation with the notion of *semantic information* in CSI, here referred as *content* of a declarative sentence *s* and denoted by 'Cont(*s*)'. In this way, the CBP consists in the evidence that, because an always true sentence is true for all possible worlds, i.e., it does not exclude any world, it is empty of any semantic content (effectively, it is a tautology), the maximum semantic content is for the always false (i.e., a contradictory) sentence, because it excludes any possible world.

In Carnap & Bar Hillel terms, "a self-contradictory sentence asserts too much: it is too informative for being true" [44, p. 229]. Effectively, it is well-known also to common-sense that tautologies have no information content. What is paradoxical for common-sense is that contradictions have the maximum information content. For logicians, however, who know the famous Pseudo-Scotus law, according to which anything can be derived from contradictions, this conclusion is not surprising, once we have defined the information content of a sentence *s*, Cont(*s*), as the set of all sentences (possible worlds) belonging to the same Universe *W* of the theory excluded by *s*.

Of course, the limit of CSI consists in its abstraction, namely in the *logical* notion of truth and on the *a priori* probability that it supposes. Surprisingly, but not contradictorily, it is just a supposition of *a logical notion of truth* (= true in all possible contexts, or "worlds" in modal logic terms) that makes impossible using truth as a necessary condition of meaningfulness in CSI.

What makes interesting the TSSI of Floridi and followers is that it offers a theory and measures of the semantic information for *contingent* and not *necessary* propositions. Namely, for propositions that are not *logically* true, i.e., true for all possible worlds, like, on the contrary, both the tautologies (i.e., the logical laws) and/or the general ontology propositions are – i.e., true for whichever "being as being". Namely, both the propositions of all empirical sciences, and the propositions of specific ontologies are *true* for objects *actually* existing (or *existed*, or that *will exist*) only in *some* possible worlds – in the limit *one*: the actual, "present" world. In other terms, the scientific and ontological theories are "models" (i.e., theories true only for a limited domain of objects), precisely because both have a semantic content, differently from tautologies. I developed elsewhere [53] a formal ontology of the QFT paradigm in natural sciences, in which this notion of truth is logically and ontologically justified, alternative to Carnap's logical atomism. I.e., alternative to the formal ontology of the Newtonian paradigm in natural sciences, on which both CSI and BCP depend.

Hence, it is highly significant developing a *theory* and a *measure* of information content such as TSSI, compatible with what S. Sequoiah-Grayson defines as the *Contingency Requirement of Informativeness* (CRI), supposed in TSSI. Unfortunately, a requirement such as CRI cannot be *supposed*, but only *justified*, as G. Dodig-Crnkovic indirectly emphasizes in her criticism to TSSI [54], and this is the limit of TSSI. In fact, the CRI states [51]: «A declarative sentence *s* is informative *iff s* individuates at least some but not all w_i from W (where $w_i \in W$)». Sequoiah-Grayson recognizes that CRI in TSSI is an idealization. However, he continues,

> Despite this idealization, CRI remains a convincing modal intuition. For a declarative sentence *s* to be informative, in some useful sense of the term, it must stake out a claim as to which world, out of the entire modal space, is in fact the actual world.

This requirement is explicitly and formally satisfied in the formal ontology of the "natural realism" as alternative to the "logical atomism" of CSI [53, 55]. Effectively the main reason, Floridi states, leading him to defend the TSSI is that only such a theory having truthfulness as *necessary condition* of meaningfulness can be useful in an epistemic logic. In it, indeed, the entire problem consists in the justification of the passage from belief as "opinion" to belief as "knowledge", intended as a *true* belief.

That in TSSI is operating a CRI it is evident from the "factual" character of the semantic information content in it, and of its probabilistic measure. Starting from the principle that semantic information σ has to be measured in terms of distance of σ from w, we have effectively four possibilities. Using the same example of Floridi [50, p. 55ff.], let us suppose that there are exactly three people in the room: this is the situation denoted in terms of the actual world w. The four possibilities for σ as to w are:

(T) There are *or* there are not people in the room;
(V) There are some people in the room;
(P) There are three people in the room;
(F) There are *and* there are not people in the room.

By defining θ as the distance between σ and w, we have: $\theta(\mathrm{T}) = 1$; $\theta(\mathrm{V}) = 0.25$ (for the sake of simplicity); $\theta(\mathrm{P}) = 0$; $\theta(\mathrm{F}) = -1$. From these relations it is possible to define the *degree of informativeness* ι of σ, that is:

$$\iota(\sigma) = 1 - \theta(\sigma)^2$$

The graph generated by the equation above (see Figure 1a) shows as θ ranges from the necessary false (F) (= contradiction), to the necessary true (T) (= tautology), both showing the maximum distance from the contingent true (P).

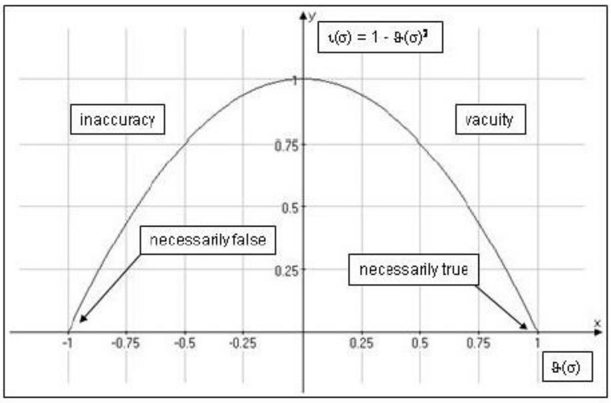


Figure 1a. Degree of informativeness. From [50, p. 56].

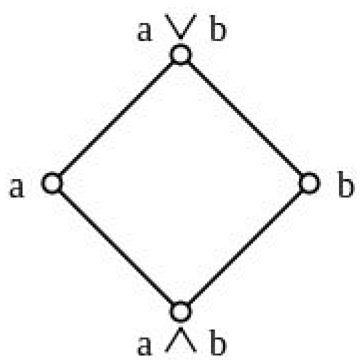


Figure 1b. Boolean lattice in equation logic

To calculate the quantity of semantic information contained in σ relative to $\iota(\sigma)$ we need to calculate the area delimited by the equation above, that is, the definite integral of the function $\iota(\sigma)$ on the interval [0, 1]. On the contrary, the amount of vacuous information, we denote as β, is also a function of θ. More precisely it is a function of the distance of θ from w, i.e.:

$$\int_0^\theta \iota(\sigma)dx = \beta$$

It is evident that in the case of (P) $\beta = 0$. From α and β, it is possible to calculate the amount of semantic information carried by σ, i.e. γ, as the difference between the maximum information that can be carried in principle by σ and by the vacuous information carried effectively by σ, that is, in bit:

$$\gamma(\sigma) = \log(\alpha - \beta)$$

Of course in the case of (P):

$$\gamma(\mathrm{P}) = \log(\alpha)$$

That confirms CRI in TSSI, that is, the proposition contingently true, namely, denoting the actual situation w and/or expressing the true knowledge of w, is carrying the maximum of semantic information about w.

5 Coalgebraic semantics of quantum systems

5.1 Category theory logic and coalgebraic semantics

To satisfy Dodig-Crnkovic's criticism about the necessity of a *formal justification* of the notion of "local (contingent) truth" theory in logic and computability theory, let us start from the extension of the Boolean lattice (matrix) of Figure 1b from the propositional calculus (Boolean equation logic) to the *monadic predicate calculus,* that is, where the proposition $b = \neg a$. In such a case, the *meet* of the lattice ($a \wedge b$) would correspond to the *always false* proposition ($a \wedge \neg a$), and the *join* ($a \vee b$) would correspond to the *always true* proposition ($a \vee \neg a$) of the quasi-probability distribution of the Figure 1a, while the maximum of this distribution corresponds to the assertion of $|a|$ (and not of $a \mid b$, as in the lattice in figure) as "locally true". To make computationally effective this representation, it is necessary that we are allowed to associate this maximum to a measure of the *maximum of entropy* expressing the "matching" (convergence till equivalence) of the results of two "concurrent computations" of a system and of its environment, as the result of the "physical work" of the phase space dynamic reconfiguration (phase transition), consuming all the available "free energy", generated by the original "mismatch" between them.

What is highly significant for our aims is that in a way completely independent from quantum physicists – at least till the very last years (see sect. 5.2 below) – logicians and computer scientists developed in the context of CT logic a coalgebraic approach to Boolean algebra semantics that only recently started to be applied also to quantum computing. Let us start from some basic notions of the CT logic (for a survey, see [56]).

The starting point of such a logic as to set theory is that the fundamental objects of CT are not "elements" but "arrows", in the sense that also the set elements are always considered as domains-codomains of *arrows* or *morphisms* – in the case of sets, domains-codomains of *functions*.

In this sense, any object *A, B, C,* characterizing a category, can be substituted by the correspondent *reflexive morphism* $A \rightarrow A$ constituting *a relation identity* Id_A. Morover, for each triple of objects, *A,B,C,* there exists a *composition map* $A \xrightarrow{f} B \xrightarrow{g} C$, written as $g \circ f$ (or sometimes: $f\,;g$), where B is the codomain of f and domain of g[4]. Therefore, a *category* is any structure in logic or mathematics with structure-preserving morphisms. E.g., in set theoretic semantics, all the models of a given formal system because sharing the same structure constitute a category. In this way, some fundamental mathematical and logical structures are as many categories: **Set** (sets and functions), **Grp** (groups and homomorphisms), **Top** (topological spaces and continuous functions), **Pos** (partially ordered sets and monotone functions), **Vect** (vector spaces defined on numerical fields and linear functions), etc.

Another fundamental notion in CT is the notion of *functor*, *F*, that is, an operation mapping objects and arrows of a category **C** into another **D**, $F\colon \mathbf{C} \rightarrow \mathbf{D}$, so to preserve compositions and identities. In this way, between the two categories there exists a *homomorphism up to isomorphism*. Generally, a functor *F* is *covariant,* that is, it preserves arrows di-

[4] We recall that typical example of function composition is a recursive, iterated function: $x_{n+1} = f(x_n)$.

rections and composition orders (e.g., in the QM attempt of interpreting thermodynamics within kinematics [57]), i.e.: if $f : A \to B$, then $FA \to FB$; if $f \circ g$, then $F(f \circ g) = Ff \circ Fg$; if id_A, then $Fid_A = id_{FA}$. However, two categories can be equally homomorphic up to isomorphism if the functor G connecting them is *contravariant*, i.e., *reversing* all the arrows directions and the composition orders, i.e. G: **C** → **D^op**:
if $f : A \to B$, then $GB \to GA$; if $f \circ g$, then $G(g \circ f) = Gg \circ Gf$; but if id_A, then $Gid_A = id_{GA}$.

Through the notion of contravariant functor, we can introduce the notion of *category duality*. Namely, given a category **C** and an *endofunctor* E: **C** → **C**, the contravariant application of E links a category to its opposite, i.e.: E^{op}: $\mathbf{C} \to \mathbf{C}^{\mathbf{op}}$. In this way it is possible to demonstrate the *dual equivalence* between them, in symbols: $\mathbf{C} \rightleftharpoons \mathbf{C}^{\mathbf{op}}$. In CT semantics, this means that given a statement α defined on **C** α is true *iff* the statement α^{op} defined on $\mathbf{C}^{\mathbf{op}}$ is also true. In other terms, truth is invariant for such an exchange operation over the statements, that is, they are *dually equivalent.* In symbols: $\alpha \rightleftarrows \alpha^{op}$, as distinguished from the ordinary equivalence of the logical tautology: $\alpha \leftrightarrow \beta$, defined within the very same category. We can anticipate here that the physical basis of this notion is precisely the *energy balance* between a system and its *thermal bath*, as far as interpreted as the *duality* between an algebra and its coalgebra, given that it is standard in modern physics to model physical systems through algebraic (and now, more effectively, coalgebraic) structures.

A particular category, indeed, that is interesting for our aims is the category of Algebras, **Alg.** They constitute a category because any algebra $\mathcal{A}$, can be defined as *a structure defined on sets* characterized by an endofunctor projecting all the possible combinations (Cartesian *products*) of the subsets of the carrier set, on which the algebra is defined, onto the set itself, that is, $\mathcal{A} \otimes \mathcal{A} \to \mathcal{A}$. The other category interesting for us is the category of coalgebras **Coalg.** Generally, a coalgebra can be defined as a structure defined on sets, whose endofunctor projects from the carrier set onto the *coproducts* of this same set, i.e., $\mathcal{A} \to \mathcal{A} \otimes \mathcal{A}$. Despite the appearances, an algebra and its coalgebra *are not dual.* This is the case, for instance, of a fundamental category of algebras in physics, that is, the *Hopf Algebras,* **Halg,** generally used in dynamic system theory both in classical and in quantum mechanics, as we know. Each *HAlg* is essentially a *bi-algebra* because including two types of operations on/to the carrier set, where – because used to represent energetically closed systems – products (algebra) and coproducts (coalgebra) can be defined on the same basis, and therefore *commute* among themselves. That is, there exists a complete *symmetry* between a *HAlg* and its *HCoalg* so that they are *equivalent* and not *dually equivalent.* In this sense any Hopf algebra is said to be *self-dual,* that is, isomorphic with itself. To make dually equivalent a Hopf algebra with its coalgebra, as we know from QFT, we have to introduce a *q*-deformation, where *q* is a thermal parameter.

More generally, indeed, it is possible to define a dual equivalence between two categories of algebras and coalgebras by a contravariant application of the same functor. This is particularly significant whereas it is meaningless that both are defined on the same basis, and therefore products and coproducts do not commute among themselves. Two are the examples that we might give of this notion, the former in mathematics and computability theory concerning Boolean algebras, the second in computational physics concerning QFT.

5.2 Coalgebraic semantics of a Boolean logic for a contravariant functor

The first example concerning Boolean algebras depends essentially of the fundamental representation theorem for Boolean algebras demonstrated in 1936 by the American mathematician M. Stone, five years after having demonstrated with John Von Neumann the fundamental theorem of QM we quoted in sect. 2. Indeed, the Stone theorem, associates each Boolean algebra B to its Stone space $S(B)$ [58]. Therefore, the simplest version of the Stone representation theorem states that every Boolean algebra B is *isomorphic* to the algebra of partially *ordered by inclusion* closed-open (clopen) subsets of its Stone space $S(B)$, effectively an *ultrafilter*[5] of the power set of a *given set (interval) of real numbers* defined on $S(B)$.

Because, each homomorphism between a Boolean algebra A and a Boolean algebra B corresponds to a continuous function from $S(B)$ to $S(A)$, we can state that each endofunctor Ω in the category of the Stone spaces, **Stone** (where the

[5] We recall here that by an "ultrafilter" we intend the maximal partially ordered set defined on the power-set of a given set ordered by inclusion, and excluding the empty set.

objects are Stone spaces and the arrows are continuous functions)**,** *induces* a contravariant functor Ω* in the category of the Boolean algebras, **BAlg** (where the objects are Boolean algebras and the arrows are recursive functions). In CT terms, the theorem states the *dual equivalence* between them, i.e., **Stone(Ω)⇌BAlg(Ω*).**

It is difficult to exaggerate the fundamental importance of the Stone theorem that, according to the computer scientists, inaugurated the "Stone era" in computer science. Particularly, this theorem demonstrated definitively that Boolean logic semantics requires only *a first-order semantics* because it requires only *partially ordered* sets and not *totally ordered sets*. This result is particularly relevant for the foundations of computability theory. Indeed, the demonstration of the fundamental Lövenheim-Skolem theorem (1921) blocked the research program of E. Schröder of the so-called "algebra of logic" in the foundations of mathematics and of calculus [59], because it demonstrated that algebraic sets are not able to deal with *non-denumerable sets*, e.g., with the *totality* of real numbers. For this reason, and the subsequent fundamental demonstrations of Tarski's theory of truth as correspondence (1929) [60], and of Gödel's incompleteness theorems (1931) [61], the set-theoretic semantics migrated to higher-order logic, so to grant the *total ordering* of sets, by some foundation axiom, e.g., the *axiom of regularity* in ZF. In this way, no *infinite chain of inclusions* among sets is allowed in *standard* set theory, so to separate the semantic "set ordering" from the complete "set enumerability"[6].

Therefore, the further step for making computationally effective the Stone theorem for a Boolean first-order semantics, avoiding the limits of the Turing-like computation scheme strictly dependent on Gödel and Tarski theorems, is the definition of *non-standard* set theories without foundation axioms. In this way, we allow infinite chains of set inclusions, according to the original intuition of the Italian mathematician E. De Giorgi [62, 63]. The most effective among the non-standard set theory is Aczel's set theory of *non-wellfounded (NWF)sets* based on the *anti-foundation axiom* (AFA) [64]. AFA, indeed, allowing set *self-inclusions* and therefore infinite chains of set inclusions, makes also possible to define the powerful notion of set *co-induction* by *co-recursion*, *dual* to the algebraic notion of *induction* by *recursion*, both as formal methods of set definition and proof [65, 66, 63] (See below Appendix 7.1).

In this sense, the key-role of the AFA axiom is threefold.

1. Before all, it grants the *compositionality* of the set inclusion relations by prohibiting that the ordinary transitivity rule (TR), $\langle \forall u,v,w \ ((uRv \wedge vRw) \rightarrow uRw) \rangle$, – where *R* is the inclusion relation and *u, v, w* are sets – holds in set inclusions, because TR supposes the set total ordering. In this way, because only the "weaker" transitivity of the Euclidean rule (ER) $\langle \forall u,v,w \ ((uRv \wedge uRw) \rightarrow vRw) \rangle$ between inclusions is here allowed, this means that the representation of sets ordered by inclusion as *oriented graphs*, in which the nodes are sets and the edges are inclusions with one only root (in our case the set *u*), satisfies *always* an "ascendant-descendant relationship" without "jumps" (each descendant has always its own ascendant, i.e., they form a *tree*). This is the core of the "compositionality" of the *inclusion operator* of a coalgebra defined on NWF sets, i.e., the basis of the so-called "tree-unfolding" of NWF sets, starting from an "ultimate root" similar to the *universal set V* – which is here allowed, because of the possibility of set self-inclusion[7] –, i.e., the disjunction of all sets forming the universe of the theory, like the "join" of a Boolean lattice. All this is the basis for extending the dual equivalence between the category **Stone** and the category **BAlg,** to the dual equivalence between the category of the coalgebras **Coalg** and the category of the algebras **Alg,** for an induced contravariant functor Ω*, i.e., **Coalg(Ω)** ⇆ **Alg(Ω*)** [67, p. 417ff.][8].
2. Secondly, the AFA axiom and the "final coalgebra theorem" justify *the coalgebraic interpretation of modal logic* in the framework of *first-order logic* (see the fundamental Van Benthem's Theorem on this regard [68]) because the principle of set unfolding for partially ordered sets within an unbounded chain of set inclusions gives us an algebraically "natural" interpretation of the modal *possibility operator* "◊", in the sense that $<\Diamond\alpha>$ means that "α is true in *some* possible worlds" [69, 70, 71, 72], so to give a computationally effective (first-order logic, where the predicate calculus is complete) justification to Thomason's early program of "reduction of the second-order logic to the modal logic" [73], made effective by another celebrated theorem, the Goldblatt-Thomason Theorem. Because any set tree

[6] Two corollaries of the Lövenhiem-Skolem theorem, demonstrated by Skolem himself in 1925 are significant for our aims, i.e., 1) that only *complete* theories are *categorical*, and 2) that the *cardinality* of an algebraic set depends intrinsically by the algebra defined on it. Think, for instance at the principle of *induction by recursion* for Boolean algebras, allowing a Boolean algebra to *construct* the sets on which its semantics is justified, blocking however Boolean computability on *finite* sets. It is evident that Zermelo's strategy of migrating to second-order set-theoretic semantics grants categoricity to mathematics on an *infinitistic* basis.

[7] Recall that set self-inclusion is not allowed for standard sets because of Cantor's theorem. This impossibility is the root of all semantic antinomies in standard set theory, from which the necessity of a second-order set-theoretic semantics ultimately derives.

[8] This depends on the trivial observation that a coalgebra $C = \langle C,\gamma: C \rightarrow \Omega C\rangle$, where γ is a transition function characterizing *C*, over an endofunctor Ω: $\mathbf{C} \rightarrow \mathbf{C}$ can be seen also as an algebra in the opposite category $\mathbf{C^{op}}$, i.e., **Coalg(Ω)** = $(\mathbf{Alg(\Omega^{op})})^{\mathbf{op}}$ [67, p. 417]

can be modeled as a Kripke *frame*, this theorem defines rigorously which elementary classes of frames are modally definable (for a deep discussion of this theorem, see [74, pp. 33-43]. For an intuitive treatment of these notions, see sect. 7.2 in the Appendix) .

3. Thirdly, in the fundamental paper of 1988 [75] Abramsky first suggested that the endofunctor of modal coalgebras is the so-called "Vietoris functor" $\mathcal{V}$ [9]. In this way we can extend the duality between coalgebras and algebras for the induction of a contravariant functor Ω*, to the *dual equivalence* between modal coalgebras and modal algebras for the induction of a contravariant functor $\mathcal{V}$*, i.e., **Coalg($\mathcal{V}$) $\leftrightharpoons$ Alg($\mathcal{V}$*)** [67, p. 393ff.]**.** This depends on the fact that $\mathcal{V}$ is a functor defined on a particular category of topological spaces, the category of the vector spaces **Vect** we introduced in sect. 5.1. Vector spaces are fundamental in physics: also the Hilbert spaces of the quantum physics mathematical formalism belong to such a category. The morphisms characterizing the vector space category are, indeed, linear functions, so if we apply to modal coalgebras Van Benthem's "correspondence theorem" [76] and the consequent "correspondence theory" [68] between the modal logic and the decidable fragments of the first order monadic predicate calculus, associating each axiom of modal calculus with a first order formula (see in Appendix 7.2 some examples), we obtain the following amazing result that Abramsky first suggested [75], and Kupke, Kurz & Venema developed [72]. Namely, we can formally justifying the modal coinduction (tree-unfolding) of predicate domains so to justify *the modal operators* of the "possible converse membership" or "possible co-membership", $\langle \ni \rangle$, and of the "actual co-membership", i.e., $\neg\langle \neg \ni \rangle$, that is, $[\ni]$, where the angular and square parentheses are reminders of the possibility-necessity, "◊-□" operators, respectively [67, p. 392ff.].

What, intuitively, all this means for our aims is that, because modal coalgebras admit only a *stratified (indexed) usage* of the necessity operator □ and of the universal quantifier ∀, since a set *actually* exists as far as effectively *unfolded* by a co-inductive procedure, the semantic evaluations in the Boolean logic effectively consist in a *convergence* between an inductive "constructive" procedure, and a co-inductive "unfolding" procedure. Namely, they effectively consist in the superposition *limit/colimit* between two concurrent inductive/coinductive computations (see Appendix 7.1). This is the core of Abramsky notion of *finitary objects* as "limits of finite ones", definable only on NWF sets, finitary objects that according to him are the most proper objects of the mathematical modeling of computations [75].

This is also the core of the related notion of *duality* between an *initial algebra*, starting from a *least* fixed-point, $x = f(x)$, and its *final coalgebra,* starting from a *greatest* fixed-point (see Appendix 7.1), at the basis of the notion of *Universal Coalgebra* as a "general theory of both computing and dynamic systems" [66]. This theory allows to justify a *formal semantics of computer programming* as satisfaction of a given program onto the physical states of a computing system, outside the Turing paradigm. Indeed, this approach systematically avoids the necessity of referring to an UTM for justifying formally the *universality* in computations, because of the possibility of referring to the algebraic and coalgebraic universality[10]. At the same time, this theory is able to give a *strong formal foundation* to the notion of *natural computation,* as far as we extend such a coalgebraic semantics to quantum systems and quantum computation. This research program has been inaugurated by S. Abramsky and his group at Oxford only few years ago, both in fundamental physics [77], and in QM computing [78], even though it has its most natural implementation in a QFT foundation of both quantum physics and quantum computation [79][11].

---

[9] The fundamental property of $\mathcal{V}$ is that it is the counterpart of the power set functor $\wp$ in the category of the topological spaces (i.e., for continuous functions) such as the Stone space category, **Stone**. This functor maps a set S to its power set $\wp(S)$ and a function $f : S \to S'$ to the image map $\wp f$ given by $(\wp f)(X) \coloneqq f[X]$ (= $\{ f(x) \mid x \in X\}$). Applied to Kripke's relational semantics in modal logic, this means that Kripke's *frames* and *models* are nothing but "coalgebras in disguise". Indeed, a *frame* is a set of "possible worlds" (subsets, s) of a given "universe" (set, S) and a binary "accessibility" relation R between worlds, $R \subseteq S \times S$. A Kripke's *model* is thus a frame with an *evaluation function* defined on it. Now R can be represented by the function $R[\bullet]: S \to \wp(S)$, mapping a point s to the collection $R[s]$ of its successors. In this way frames in modal logic correspond to coalgebras over the *covariant* power set functor $\wp$. For such a reconstruction see [67, p. 391].

[10] However, see the fundamental remarks about the limits of *decidability* and *computability* in this first-order modal logic semantic approach in [74], in which it is said, just in the conclusion, that one of the most promising research program in this field is related with the coalgebraic approach to modal logic semantics.

[11] This depends on the fact that *contravariance* in QM algebraic representation theory can have only an *indirect justification*, as Abramsky elegantly explained in his just quoted paper. QM algebraic formalism is, indeed, intrinsically based on Von Neumann's *covariant* algebra, so that only Hopf algebras' self-duality are "naturally" (in the algebraic sense of the allowed functorial transforms) justified in it [57, 89, 90].

From the standpoint of the *natural ontology* of cognitive neuro-dynamics in the framework of a QFT foundation of it (see sect. 3.2 above), all this, roughly speaking, means that it is *logically* true that the *(sub-)class* of horses is a *member* of the *(super-)class* of mammalians *iff, dually,* it is *ontically* (dynamically) true that a *co-membership* of the *species* of horses to the *genus* of mammalians occurs, from some step *n* onward of the universe evolution (= "natural unfolding" of a biological evolution tree). I.e.,

$$\Box_{\forall n(n>m)}\left(\underbrace{horse \in mammmalian}_{\text{Algebra}(\Omega^*)}\underbrace{\overset{\Omega^*\leftarrow\Omega}{\rightleftarrows}}_{\text{Onto-logical } iff}\underbrace{horse \ni mammalian}_{\text{Co-Algebra}(\Omega)}\right)$$

In other terms, we are faced here with an example of a "functorially induced" homomorphism, from a coalgebraic *natural structure* of *natural kinds* (genera/species) into a *logical structure of predicate domains* (class/sub-class), as an example of modal *local truth*, applied to a theory of the ontological natural realism, in the framework of an evolutionary cosmology [80, 53][12], where it is nonsensical to use not indexed (absolute) modal operators and quantifiers, given that physical laws emergence depends on the universe evolution. In parenthesis, this gives also a solution to the otherwise unsolved problem, in Kripke's relational semantics, of the denotation of *natural kinds* (the denoted objects of common names, such as "horses" or "mammalians" in our example) and of the connected Kripke's and Putnam's *causal theory of reference* (see on this point my previous discussion about these problems in [22]). Finally, this gives a logical interpretation as *predicate* (e.g., "being horse") of the "doubled number", i.e., $\mathcal{N}_A, \mathcal{N}_{\tilde{A}}$, as identity functions relative to two mirrored (doubled) sets of degrees of freedom, A and $\tilde{A}$, one relative to a logical realm (the Algebra(Ω*)), the other to its dual natural realm (the Coalgebra(Ω)), the latter satisfying (making true) *naturally* – i.e., *dynamically* in this QFT implementation – the former (see above, sect. 3.2). The co-membership relation in the coalgebraic half has its physical justification in QFT by the general principle of the "*foliation* of the QV" at the ground state, and of the relative Hilbert space into physically inequivalent subspaces", allowing "the building up" via SSB of ever more complex phase coherence domains in the QV, given their stability in time. They do not depend, indeed, on any energetic input (they depend on as many NGB condensates $|\mathcal{N}|$, each correspondent to a SSB of the QV at the ground state), but on as many "entanglements" with stable structures of the environment [81]. This justifies Freeman and Vitiello in suggesting that this is the fundamental mechanism of the formation of the so called "long-term memory" traces in brain dynamics [41], i.e., the formation of the "deep beliefs" in our brains by which each of us interprets the world, based on her/his past experience, using the AI recent diffused jargon in the artificial neural network computing [82].

Anyway, apart from this "ontological" exemplification, useful however to connect the present discussion with the rest of this paper, all this means extending to a Boolean lattice *L* of the monadic predicate logic the modal semantics notions of co-induction and/or of "tree unfolding", so to give the formal justification of the modal notion of "local truth" also in a computational environment [13]. Indeed, because such a co-inductive procedure of predicative domains justification is defined on NWF sets supporting set self-inclusion, i.e., $x\rightarrow\{x\}$, for each of these co-induced domains also the relative Id_x, i.e., the relative predicate φ is defined, without any necessity of referring to Fregean second-order axioms, such as the comprehension axiom of ZF set-theory, i.e.: $<\forall x\exists y\ x\in y \equiv \varphi x>$. This justifies the general statement that in CT coalgebraic semantics there exists a Tarski-like model theory [56], without, however, the necessity of referring to higher order languages for justifying the semantic meta-language [83], according to Thomason's reduction program.

We can thus conclude this section by affirming that the previous discussion satisfies the first requirement of Dodig-Crnkovic criticism to a theory of semantic information at the end of the sect. 4.2.2. I.e., the necessity of a *formal justification* of the theory of "local truth", essential for the notion and measure of Floridi's *semantic information* that can be naturally given in the context of a coalgebraic (modal) semantics of predicate logic. Quoting the first concluding remark of V. Goranko and M. Otto contribution to the *Handbook of modal logic* devoted to model theory of modal logic [84, p. 323], we can conclude too:

> Modal logic is local. Truth of a formula is evaluated at a current state (possible world); this localization is preserved (and carried) along the edges of the accessibility relations by the restricted, relativized quantification corresponding to the (indexed) modal operators.

---

[12] On this regards, the Aristotelian famous statement synthesizing his "intentional" approach to epistemology – "not the stone is in the mind, but the form of the stone" – has an operational counterpart into the homomorphism algebra-coalgebra of the QFT neuro-dynamics.

[13] This result has been recently formally obtained [88]. For an intuitive explanation of this result, see below the two Appendices, sects. 7.1-7.2.

5.3 Coalgebraic semantics of quantum systems

We have now only a last step to perform: implementing the theory of local truth in a QFT system. That is, for demonstrating that the curve of the quasi-probability diagram of Figure 1a as an information measure of the degree of semantic informativeness represents a measure of maximal entropy, expressing the fact that a given *dynamic cognitive system* (e.g., a brain dynamics in the QFT interpretation depicted in sect. 3.2) consumed all the *free-energy* deriving from the mismatch with its thermal bath, for the re-organization "work" (in the thermodynamic sense) of its inner state, so to match with it, and hence minimizing the free-energy of the whole system (brain + thermal bath). In other terms, we have to interpret the maximal entropy physical measure as a logical measure of *maximal local truth* in the statistical sense. To sum up, we have to interpret consistently a QFT dynamic system as a *computing system.*

In the light of the precedent discussion it is necessary and sufficient for such an aim to demonstrate that the collections of the "*q*-deformed Hopf algebras" and the "*q*-deformed Hopf coalgebras" of the QFT mathematical formalism constitute two *dually equivalent categories* for the contravariant application of the same functor *T*, that is, the contravariant application of the so-called *Bogoliubov transform.* This is the classical QFT operator of "particle creation-annihilation", where the necessity of such a contravariance depends on the constraint of satisfying anyway the *energy balance principle.* I.e., ***q*-HAlg(*T*)** $\rightleftharpoons$ ***q*-HCoalg(*T**)**. It is useful to recall here that the *q-deformation* parameter characterizing each pair of *q*-deformed Hopf algebra-coalgebra is physically a *thermal parameter,* so to constitute the "evolution parameter" of the universe in a QFT interpretation of cosmology, via SSB's of the QV, according to Wheeler's suggestion with which we started our paper that in the new physics paradigms the "cosmogony is the legislator of physics".

On the other hand, mathematically this parameter is related with the "Bogoliubov's angle", θ, characterizing each different application of the transform – where, as we know, the angle, with the frequency and the amplitude are the three main parameters characterizing generally the phase of a given waveform. For a systematic presentation of the QFT mathematical formalism, see [11, pp. 185-235].

The complete justification of a coalgebraic interpretation of this mathematical formalism is given elsewhere [79], because we cannot develop it here. Nevertheless, at least two points of such a justification are important to emphasize, for justifying the interpretation of the maximal entropy in a QFT system as a semantic measure of information, i.e., as a statistical measure of *maximal local truth* in a CT coalgebraic logic for QFT systems.

Firstly, the necessary condition to be satisfied in order that a coalgebra category for some endofunctor Ω, i.e., **Coalg(Ω),** can be interpreted as a *dynamic* and/or *computational* system, is that it satisfies the formal notion of *state transition system* (STS). Generally a STS is an abstract machine characterized as a pair $(S, \rightarrow)$, where *S* is a set of states, and $((\rightarrow) \subseteq S \times S)$ is a transition binary relation over *S*. If *p*, *q* belong to *S*, and (*p*, *q*) belongs to $(\rightarrow)$, then $(p \rightarrow q)$, i.e., there is a transition over *S.* For allowing that a dynamic/computational system be represented as a STS on a functorial coalgebra for some functor Ω it is necessary that the functor admits a *final coalgebra* [67, p. 389]. I.e.:

Definition 1: (Definition of final coalgebra for a functor). A functor $\Omega : \mathbf{C} \rightarrow \mathbf{C}$ is said to admit a final coalgebra iff the category **Coalg(Ω)** has a final object, that is, a coalgebra $\mathbb{Z}$ such that from every coalgebra $\mathbb{A}$ in **Coalg(Ω),** there exists a unique homomorphism, $!\mathbb{A} : \mathbb{A} \rightarrow \mathbb{Z}$.

This property has a very intriguing realization – and this is the sufficient condition to satisfy for formalizing a QFT system as a computing system – into the final coalgebra associated with a particular abstract machine, the so-called "infinite state black-box machine" $\mathbb{M}\langle M,\mu\rangle$ [67, p. 395]. It is characterized by the fact that the machine internal states, x_0, x_1, …, cannot be directly observed, but only some their values ("colors", c_n) associated with a state transition μ. I.e., $\mu(x_0) = (c_0, x_1)$, $\mu(x_1) = (c_1, x_2)$, … In this way, the only "observable" of this dynamics is the infinite sequence of behaviors or *stream beh* $(x_0) = (c_0, c_1, c_2, \ldots) \in C^{\omega}$ of value combinations or "words" over the data set *C*. The collection C^{ω} forms a *labeled* STS for the functor $C \times \mathcal{I}$, where $\mathcal{I}$ is the set of all the identity functions (labels), as far as we endow C^{ω} with a transition structure γ splitting a stream $u = c_0c_1c_2, \ldots$ into its "head" $h(u) = c_0$, and its *tail* $t(u) = c_1c_2c_3\ldots$. If we pose $\gamma(u) = (h(u),t(u))$, it is possible to demonstrate that the behavior map $x \mapsto beh(x)$ is the unique homomorphism from $\mathbb{M}$ to this coalgebra $\langle C,\gamma\rangle$, that is the final coalgebra $\mathbb{Z}$ in the category **Coalg($C \times \mathcal{I}$)**[14]**.**

The abstract machine $\mathbb{M}$ is used in TCS for modeling the *coalgebraic semantics* of programming relative to infinite data sets – the so-called *streams*: think, for instance, at internet and more generally at all the ever-growing databases ("big data") [66]. The application of $\mathbb{M}$ for characterizing the QFT dynamics as a "computing dynamics" is evident in the light of the precedent discussion because we are allowed to interpret the thermodynamic functor T (Bogoliubov transform) characterizing the category ***q*-HCoalg(*T*)** as a functor able to associate the observable c of each "word" (phase coherence domain) of the QFT infinite dataset C, i.e., the infinite CCR's characterizing the QV, with the correspondent I_c, so that $T = (C \times \mathcal{I})$. Indeed, each I_c corresponds in the QFT formalism to the NGB condensate numerical value $|\mathcal{N}|$ identifying univocally each phase coherence domain, i.e. a "word" of the QV "language". In this way, the QV, because endowed with the SSB state-transition – effectively a phase-transition – structure γ, selecting every time one CCR (*head*) as to the rest of the others (*tile*), corresponds to the final coalgebra $\mathbb{Z}$ of the category ***q*-HCoalg(*T*)**. Moreover, the dynamics of the $\mathbb{M}_{\mathrm{QFT}}$ is a *thermo*-dynamics, i.e., its state (phase) transition is "moved" by the II Principle (energy equipartition), in a way that must satisfy, on one hand, the "energy arrow contravariance" related to the I Principle, and, on the other one, without consuming all the QV energy "reservoir" as requested by the III Principle[15]. All this implies the necessity of doubling the behavior map, i.e., $x \mapsto beh(x,\tilde{x})$, and all the related objects and structures – i.e., the necessity of "echoing" each word of the QV language –, so to satisfy finally the "dual equivalence" characterizing the QFT categorical formalism, i.e., ***q*-HAlg(*T*)** $\rightleftharpoons$ ***q*-HCoalg(*T*** ***)**. In logical terms, the functor induction $T \leftarrow T^*$ means that the semantics (coalgebra) induces its own syntax (algebra). This, if justifies, on one hand, the computer scientist interest toward a coalgebraic approach to quantum computation for managing streams, on the other one, it demonstrates that the QFT interpretation of this approach is the more promising one. In fact, what we intended using the metaphor of the "word echoing" within the model of the $\mathbb{M}_{\mathrm{QFT}}$ is effectively the DDF principle determining the *dynamic choice,* observer-independent, of the structure (syntax) of the "composed Hilbert space" of a QFT system as based on the *dual equivalence* (semantics) of one pair q-HAlg(T) $\rightleftharpoons$ q-HCoalg(T^*) representing the system.

All this is related, with the second, final, observation, justifying the interpretation of the maximal entropy in a QFT "doubled" system as a semantic measure of information, i.e., as a statistical measure of *maximal local truth* in the CT coalgebraic logic. In the QFT mathematical formalism this maximum of the entropy measure is formally obtained when the above illustrated DDF principle (far from equilibrium energy balance) between a system (algebra) and its thermal bath (coalgebra) is *dynamically* (= automatically) satisfied. This means that we are allowed to interpret the QFT *qubit* of such a natural computation as an "evaluation function" in the semantic sense. Indeed, in the QFT "composed Hilbert space" including also the thermal bath degrees of freedom, $\tilde{A}$, i.e. $\mathcal{H}_{A,\tilde{A}} = \mathcal{H}_A \otimes \mathcal{H}_{\tilde{A}}$, for calculating the static and dynamic entropy associated with the time evolution generated by the free energy, i.e., $|\phi(t)\rangle, |\psi(t)\rangle$, of the qubit mixed states $|\phi\rangle, |\psi\rangle$, one needs to double the states by introducing the tilde states $|\tilde{0}\rangle$ and $|\tilde{1}\rangle$, relative to the thermal bath, i.e., $|0\rangle \to |0\rangle \otimes |\tilde{0}\rangle$, and $|1\rangle \to |1\rangle \otimes |\tilde{1}\rangle$. This means that such a QFT version of a qubit implements effectively the CNOT (controlled NOT) logical gate, which flips the state of the qubit, conditional on a *dynamic* control of an effective input matching [85, 11, pp. 91-95].

---

[14] In parenthesis, in the machine $\mathbb{M}$ the general coalgebraic principle of the *observational* (or *behavioral*) *equivalence* among states holds in the following way. Indeed, for every two coalgebras (systems) $\mathbb{S}_1, \mathbb{S}_2 \in \mathbf{Coalg}(C \times \mathcal{I})$, $(!c_{\mathbb{S}_1} = !c_{\mathbb{S}_2}) \Rightarrow (!x_{\mathbb{S}_1} = !x_{\mathbb{S}_2})$. All the scholars agree that this has an immediate meaning for quantum systems logic and mathematics, as a further justification for a coalgebraic interpretation of quantum systems.

[15] A condition elegantly satisfied in the QFT formalism by the *fractal structure* of the systems phase space and, therefore, by the *chaotic character* of the macroscopic trajectories (phase transitions) defined on it, generally [91], and specifically in the dissipative brain dynamics [26]

6 Conclusion

I used many times in this paper, also in the title, the expression “new paradigm”. Th. Kuhn who coined the fortunate expression “paradigm shift” said that the “scientific community” has to decree this shift every time it happens in the history of science. Therefore, if we agree in recognizing to the Stockholm Royal Academy the honor and the duty of representing the scientific community at its higher levels, it decreed just few months ago that we are living one of these turns characterizing the history of modern science. In the official conference press for announcing to the world that the 2015 Nobel Prize in Physics was awarded to the physicists T. Kajita and A. B. McDonald for their observational discovery of the neutrino mass, the Academy stated that “the new observations had clearly showed that the Standard Model cannot be the complete theory of the fundamental constituents of the universe” [86].

In this paper we defended the idea that the QFT interpreted as a “thermal field theory” is a candidate for constituting, before all, the proper theory of the “physics beyond the Standard Model” because able to give physics a strong formal alternative to the “perturbative methods” and their “asymptotic states” that is at the basis of the Standard Model “mechanistic” interpretation of the statistical distinction between “fermions” and “bosons”, in terms of “particles” and of “force field quanta”, respectively. The validity of perturbative methods relies indeed on the possibility of correctly defining asymptotic states for the system, namely states defined in infinitely distant space-time regions, so to make interactions negligible, in the presumption that this representation is not falsifying the nature of the physical system to be represented. In this light, the “paradigm turn” with which today we are faced is therefore not only with respect to the Standard Model physics (QED and QCD, i.e., the so-called “standard QFT”), but also as to the QM, and as to the “many body dynamics” extension of the Classical Mechanics, from Laplace on. Therefore, the presumption of correctly representing a system as isolated lies at the bottom of the same origins of the modern physics and of the modern calculus. This presumption cannot hold, however, in the case of QFT systems as intrinsically “open” to the background QV fluctuations, or, more generally, when we have to reckon with system phase transitions. In all these cases, the QFT alternative picture of representing both fermions and bosons as quanta of the relative force field is the more suitable. On the other hand, we showed that the paradigm shift we are discussing, because involving the foundations of modern sciences, involves also the foundations of mathematics and of the computability theory, as far as both related with non-standard set theories.

The alternative formalism offered by the thermal QFT is, therefore, the doubled algebra representation of a quantum system and of its thermal bath, through the mathematical formalism of the DDF between a q-deformed Hopf algebra and its q-deformed Hopf coalgebra, illustrated in this paper. Such formalism has been successfully applied not only in fundamental physics, the physics of the neutrino oscillations included [11, pp. 91-95], but also in condensed matter physics, the biological matter and the brain dynamics included. For this reason, in this paper we deepened the possibility of justifying in such a formalism also the notion and measure of “semantic information”, generally associated with the biological and neural information processing. For this aim, we discussed an information theoretic interpretation of the DDF in QFT systems, in the framework of the coalgebraic approach to quantum computation, recently introduced as an alternative to the information theoretic interpretation of QM systems as Quantum Turing Machines. This allowed us to give a formal justification of the notion of “local truth”, associated to the measure of semantic information that is therefore interpreted as a measure of maximal entropy, because minimization of the “free energy” associated to the mismatch with the system environment thermal bath. Practically, this measure expresses the “entanglement” dynamically occurred, and signaled by the flipping of the associated qubit, between the degrees of freedom of the system and of its thermal bath, within the same representation space including both. This result is ultimately based on the possibility of justifying the dual equivalence between the categories of the q-deformed Hopf algebras and the q-deformed Hopf coalgebras, allowing to interpret quantum computations of QFT systems in the framework of Category Theory logic. This initial result opens the way to new promising scenarios in quantum natural and artificial computation to be explored in the next future.

7 Appendixes

7.1 Induction and coinduction as principles of set definition and proof for Boolean lattices

The collection of clopen subsets of a Stone space, as to which a Boolean algebra is isomorphic, according to the Stone theorem is effectively an ultrafilter U (or the maximal filter F) on the power-set, $\wp(S)$, of the set S. Namely, it is the *maximal partially ordered set* (maximal poset) within $\wp(S)$ ordered by inclusion, i.e., $(\wp(S), \subseteq)$, with the exclusion of the empty set. Any filter F is *dual* to an *ideal I*, simply obtained in set (order) theory by inverting all the relations in F, that is, $x \leq y$ with $y \leq x$, and by substituting *intersections* with *unions*. From this derives that each ultrafilter U is dual to a greatest ideal that, in Boolean algebra, is also a *prime* ideal, because of the so-called *prime ideal theorem*, effectively a corollary of the Stone theorem, demonstrated by himself. All this, applied to the Stone theorem, means that the collection of partially ordered clopen subsets of the Stone space to which a Boolean algebra is isomorphic, corresponds to a Boolean logic complete lattice L for a *monadic first order predicate logic*. From this, the definition of *induction* and *coinduction* as dual principles of set definition and proof is immediate, as soon as we recall that the fixed-point of a computation F is given by the equality $x = F(x)$ [63, p. 46]:

Definition 2 (sets inductively/co-inductively defined by F). For a complete Boolean lattice L whose points are sets, and for an endofunction F, the sets

$$F_{ind} := \bigcap \{x \mid F(x) \leq x\}$$
$$F_{coind} := \bigcup \{x \mid x \leq F(x)\}$$

are, respectively, the sets *inductively* defined by a *recursive F*, and *co-inductively* defined by a *co-recursive F*. They correspond, respectively, to the *meet* of the pre-fixed point and the *join* of the post-fixed points in the lattice L, i.e., the least and greatest fixed-points, if F is monotone, as required from the definition of the category **Pos** (see above, sect. 5.1).

Definition 3 (induction and co-induction proof principles). In the hypothesis of **Definition 2**, we have:

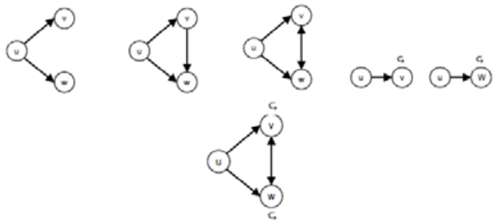
$$\textit{if } F(x) \leq x \textit{ then } F_{ind} \leq x \qquad \text{(induction as a method of proof)}$$
$$\textit{if } x \leq F(x) \textit{ then } x \leq F_{coind} \qquad \text{(co-induction as a method of proof)}$$

These two definitions are the basis for the *duality* between an *initial algebra* and its *final coalgebra*, as a new paradigm of computability, i.e., Abramsky's *finitary* one, and henceforth for the duality between the *Universal Algebra* and the *Universal Coalgebra* [66].

7.2 The extension of coinduction method to the definition of a complete Boolean Lattice of monadic predicates

The fundamental result of the above quoted Goldblatt-Thomason Theorem and Van Benthem Theorem is that a set-tree of NWF sets – effectively a set represented as an oriented graph where nodes are sets, and edges are inclusion relations with subsets governed by an Euclidean rule – corresponds to the structure of a Kripke *frame* of his relational semantics, characterized by a set of "worlds" and by a two-place accessibility relation R between worlds. E.g., the second graph from left below corresponds to the graph of the number 3, with $u = 3$, $v = 2$, $w = 1$. Therefore for understanding intuitively the extension of the coinduction method to the domains of monadic predicates of a Boolean lattice, let us start from 1) the "Euclidean rule (ER)" $\langle \forall u,v,w \; ((uRv \wedge uRw) \rightarrow vRw) \rangle$ (see the second from left graph below), driving all the NWF set inclusions and that is associated by Van Benthem's Correspondence Theorem to the modal axiom **E** (or **5**): $\langle \Diamond\alpha \rightarrow \Box\Diamond\alpha \rangle$, of the modal propositional calculus, and 2) from the "seriality rule (SR)" $\langle \forall u \exists v \; (uRv) \rangle$ (an example of this axiom is given by the fourth or the fifth graph below) – that has an immediate physical sense, because it corresponds to whichever energy conservation principle in physics, e.g., the I Principle of Thermodynamics –, and that is associated to the modal axiom **D:** $\langle \Box\alpha \rightarrow \Diamond\alpha \rangle$. The straightforward first order calculus, by which it is possible formally justifying the definition/justification by co-induction (tree unfolding) of an *equivalence class* as the domain of a given monadic predicate, through the application of the two above rules to whichever triple of objects $\langle u,v,w \rangle$, is the following.

For ER, $\langle \forall u,v,w\ ((uRv \wedge uRw) \rightarrow vRw) \rangle$; hence, for seriality, $\langle \forall u,v\ (u\mathrm{R}v \rightarrow v\mathrm{R}v) \rangle$; finally: $\langle \forall u,v,w\ [((uRv \wedge uRw) \rightarrow (vRw \wedge wRv \wedge vRv \wedge wRw)) \leftrightarrow ((v \equiv w) \subset u)] \rangle$. I.e., $(v \equiv w)$ constitutes an equivalence class, say **Y**, because a "generated" transitive[16]-symmetric-reflexive relation holds among its elements, which are therefore as many "descendants" of their common "ascendant", *u*. More intuitively, using Kripke's relational semantics graphs for modal logics, where $\langle u,v,w \rangle$ are as many "possible worlds" (models) of a given universe *W*, and where *R* is the two-place "accessibility relation" between worlds, the above calculus reads:



The final graph constitute a Kripke-like representation of the **KD45** modal system, also defined in literature as "secondary **S5**", since the equivalence relationship among all the possible worlds characterizing **S5** here holds only for a subset of them, that . In our example, the subset of worlds $\{w,v\}$.

REFERENCES

[1] C. M. Patton e J. A. Wheeler, «Is physics legislated by cosmogony?,» in *Quantum gravity*, C. J. Isham, R. Penrose e D. W. Sciama, A cura di, Oxford, Clarendon Press, 1975, pp. 538-605.

[2] R. Feynman, «Simulating physics with computers,» *Int. J. Theor. Phys.*, vol. 21, p. 467–488, 1982.

[3] J. A. Wheeler, «Information, physics, quantum: The search for links,» in *Complexity, entropy, and the physics of information*, W. H. Zurek, A cura di, Redwood City, CA, Addison-Wesley, 1990.

[4] D. Deutsch, «Quantum theory, the Church-Turing principle and the universal quantum computer,» *Proc. R. Soc. Lond. A*, vol. 400, p. 97–117, 1985.

[5] C. Rovelli, «Relational quantum mechanics,» *Int. J. Theor. Phys.*, vol. 35, p. 1637–1678, 1996.

[6] H. Zenil, Ed., A computable universe. Understanding and exploring nature as computation. Foreword by Sir Roger Penrose, Singapore-Hackensack, NJ-London: World Scientific Publishing, 2013.

[7] C. Fields, «If Physics Is an Information Science, What Is an Observer?,» *Information*, vol. 3, n. 1, pp. 92-123, 2012.

[8] H. D. Zeh, «Wave function: 'it' or 'bit'?,» in *Science and Ultimate Reality*, J. D. Barrow, P. C. W. Davies e C. L. Harper Jr., A cura di, Cambridge, MA, Cambridge UP, 2004, pp. 103-120.

[9] H. D. Zeh, «Quantum discreteness is an illusion,» *Foundations of Physics*, vol. 40, pp. 1476-1493, 2010.

[10] M. Tegmark, «How unitary cosmology generalizes thermodynamics and solves the inflactionary entropy problem,» 2011. [Online]. Available: http://arxiv.org/pdf/1108.3080.pdf. [Consultato il giorno 16 March 2012].

[11] M. Blasone, P. Jizba e G. Vitiello, Quantum field theory and its macroscopic manifestations. Boson condensation, ordered patternsand topological defects, London: Imperial College Press, 2011.

[12] E. Del Giudice, R. Pulselli e E. Tiezzi, «Thermodynamics of irreversible processes and quantum field theory: an interplay for understanding of ecosystem dynamics,» *Ecological Modelling*, vol. 220, pp. 1874-1879, 2009.

[13] J. Von Neumann, Mathematical foundations of quantum mechanics, Princeton, NJ: Princeton UP, 1955.

[14] J. Goldstone, «Goldstone, J (1961). "Field Theories with Superconductor Solutions". 19:,» *Nuovo Cimento*, vol. 19, p. 154–164, 1961.

[15] J. Goldstone, A. Salam e S. Weinberg, «Broken Symmetries,» *Physical Review*, vol. 127, p. 965–970, 1962.

[16] G. Vitiello, «Links. Relating different physical systems through the common QFT algebraic structure,» *Lecture Notes in Physics*, vol. 718, pp. 165-205, 2007.

[17] Y. Nambu, «Quasiparticles and Gauge Invariance in the Theory of Superconductivity,» *Physical Review* , vol. 117, p. 648–663, 1960.

[18] C. Itzykson e . J. Zuber, Quantum field theory, New York: McGraw-Hill, 1980.

[19] H. Umezawa, Advanced field theory: micro, macro and thermal concepts, New York: American Institute of Physics, 1993.

[16] Remember that the transitive rule in the NWF set theory does not hold only for the inclusion operation, i.e., for the superset/subset ordering relation.

[20] H. Umezawa, «H. Umezawa, Development in concepts in quantum field theory in half century,» *Math. Japonica,* vol. 41, p. 109–124, 1995.

[21] G. Vitiello, «The dissipative brain,» in *Brain and Being - At the boundary between science,philosophy,language and arts*, G. G. Globus, K. H. Pribram e G. Vitiello, A cura di, Amstedam, John Benjamins Pub. Co., 2004, pp. 317-330.

[22] G. Basti, «Intelligence and reference. Formal ontology of the natural computation,» in *Computing Nature. Turing Centenary Perspective*, G. Dodig-Crnkovic e R. Giovagnoli, A cura di, Berlin-Heidelberg, Springer-Verlag, 2013a, pp. 139-159.

[23] G. Basti, «A change of paradigm in cognitive neurosciences Comment on: "Dissipation of 'dark energy' by cortex in knowledge retrieval" by Capolupo, Freeman and Vitiello,» *Physics of life reviews*, vol. 5, n. 10, pp. 97-98, 2013b.

[24] W. J. Freeman e G. Vitiello, «Nonlinear brain dynamics as macroscopic manifestation of underlying many-body field dynamics,» *Physics of Life Reviews,* vol. 3, n. 2, pp. 93-118, 2006.

[25] W. J. Freeman e G. Vitiello, «Dissipation and spontaneous symmetry breaking in brain dynamics,» *Journal of Physics A: Mathematical and Theoretical,* vol. 41, n. 30, p. 304042, 2008.

[26] G. Vitiello, «Coherent states, fractals and brain waves,» *New Mathematics and Natural Computing,* vol. 5, n. 1, pp. 245-264, 2009.

[27] W. J. Freeman, «Origin, structure, and role of background EEG activity. Part 1. Analytic amplitude,» *Clin. Neurophysiol.,* vol. 115, p. 2077–2088., 2004.

[28] W. J. Freeman, «Origin, structure, and role of background EEG activity. Part 2. Analytic phase,» *Clin. Neurophysiol.,* vol. 115, pp. 2089-2107, 2004.

[29] W. J. Freeman, «Origin, structure, and role of background EEG activity. Part 3. Neural frame classification,» *Clin. Neurophysiol.,* vol. 116, pp. 111-1129, 2005.

[30] W. J. Freeman, «Origin, structure, and role of background EEG activity. Part 4. Neural frame simulation,» *Clin. Neurophysiol.,* vol. 117, pp. 572-589, 2006.

[31] W. J. Freeman, B. C. Burke, M. D. Holmes e S. Vanhatalo, «Spatial spectra of scalp EEG and EMG from awake humans,» *Clin. Neurophysiol.,* vol. 114, pp. 1055-1060, 2003.

[32] W. J. Freeman , G. Ga'al e R. Jornten, «A neurobiological theory of meaning in perception. Part 3. Multiple cortical areas synchronize without loss of local autonomy,» *Intern. J. Bifurc. Chaos,* vol. 13, p. 2845–2856, 2003.

[33] W. J. Freeman e L. J. Rogers, «A neurobiological theory of meaning in perception. Part 5. Multicortical patterns of phase modulation in gamma EEG,» *Int. J. Bifurc. Chaos*, vol. 13, pp. 2867-2887, 2003.

[34] E. Schrödinger, What is life, Cambridge: Cambridge UP, 1944.

[35] A. Szent-György, An introduction to sub-molecular biology, New York, 1960.

[36] H. Frölich, «Long range coherence and energy storage in biological systems,» *Int. J. of Quantum Chemistry,* vol. 2, p. 641ff., 1968.

[37] H. Frölich, A cura di, Biological coherence and response to external stimuli, Berlin: Springer, 1988.

[38] M. W. Ho, «What is (Schrödinger's) negentropy,» *Modern Trends in BioThermoKinetics,* vol. 3, pp. 50-61, 1994.

[39] L. Szilard, «On the decrease of entropy content in a thermodynamical system by the intervention of intelligent beings,» *Behavioral Science,* vol. 9, n. 4, pp. 301-10, 1964.

[40] L. Brillouin, Science and Information Theory, New York: Academic Press, 1962.

[41] A. Capolupo, W. J. Freeman and G. Vitiello, "Dissipation of dark energy by cortex in knowledge retrieval," *Physics of life reviews,* vol. This Issue, 2013.

[42] C. E. Shannon, The mathematical theory of communication, Urbana: Univ. of Illinois Press, 1949.

[43] D. M. MacKay, Information, mechanism, and meaning, MIT Press, 1969.

[44] R. Carnap e Y. Bar-Hillel, «An outline of a theory of semantic information,» in *Languange and information: selected essays on their theory and application,* Reading, Ma & London, UK, Addison-Wesley, 1964, pp. 221-274.

[45] M. Barbieri, Semantic codes. An introduction to semantic biology, Cambridge: Cambridge UP, 2003.

[46] T. W. Deacon, Incomplete nature. How mnd emerged from matter, New York: Norton & Co., 2011.

[47] H. Atmanspacher and H. Scheingraber, "Pragmatic information and dynamical instabilities in multimode continuous-vawe dye laser," *Can. J. Phys.,* vol. 68, pp. 728-737, 1990.

[48] K. E. Cahill and R. J. Glauber, "Density operators and quasiprobability distributions," *Physical Review,* vol. 177, no. 5, pp. 1882-1902, 1969.

[49] A. N. Kolmogorov, Foundations of the theory of probability. Second English edition., New York: Chelsea Publishing, 1956.

[50] L. Floridi, «Semantic conceptions of information,» in *Stanford Encyclopedia of Philosophy. Spring 2011 Ed.,* E. N. Zalta, A cura di, 2011, pp. 1-70.

[51] S. Sequoiah-Grayson, «The metaphilosophy of information,» *Minds and Machines,* vol. 17, n. 3, pp. 331-344, 2007.

[52] R. Carnap, Meaning and Necessity: A Study in Semantics and Modal Logic, Chicago: Chicago UP, 1956.

[53] G. Basti, "From formal logic to formal ontology. The new dual paradigm in natural sciences," in *Proceedings of 1st CLE Colloquium for*

Philosophy and History of Formal Sciences, Campinas, 21-23 March 2013, Campinas-Rome, 2016.

[54] G. Dodig-Crnkovic, «System Modelling and Information Semantics,» in *Proceedings of the Fifth Conference for the Promotion of Research in IT*, J. Bubenkojr e e. al., A cura di, Lund, New Universities and University Colleges in Sweden, Studentlitteratur, 2005.

[55] G. Basti, The formal ontology of the natural realism, Campinas: Sociedade Brasileira de Historia de Matematica, 2014b.

[56] S. Abramsky e N. Tzevelekos, «Introduction to categories and categorical logic,» in *New structures for physics. Lecture Notes in Physics, 813*, B. Coecke, A cura di, Berlin-New York, Springer, 2011, pp. 3-94.

[57] A. Connes e C. Rovelli, «Von Neumann algebra automorphisms and time-thermodynamics relation in general covariant quantum theories,» *Classical and Quantum Gravity,* vol. 11, pp. 2899-2918, 1994.

[58] M. H. Stone, «The theory of representation for Boolean algebras,» *Transactions of the American Mathematical Society,* vol. 40, pp. 37-111, 1936.

[59] E. Schröder, Vorlesungen über die Algebra der Logik. (Exakte Logik). I Band, Leipzig: B. G.Teubner, 1890.

[60] A. Tarski, «The Concept of Truth in Formalized Languages,» in *Logic, Semantics, Metamathematics*, 2 a cura di, J. Corcoran, A cura di, Indianapolis, Hackett, 1983, 1935, p. 152–278.

[61] K. Gödel, «Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme, I,» *Monatshefte für Mathematik und Physik,* vol. 38, p. 173–98, 1931.

[62] M. Forti e F. Honsell, «Set Theory with Free Construction Principles,» *Scuola Normale Superiore — Pisa Classe de Scienza. . Serie IV,* vol. 10, p. 493–522, 1983.

[63] D. Sangiorgi , «Origins of bisimulation and coinduction,» in *Advanced topics in bisimulation and coinduction*, D. Sangiorgi e J. Rutten, A cura di, Cambridge, UK, Cambridge UP, 2012, pp. 1-37.

[64] P. Aczel, «Non-well-founded sets,» *CLSI Lecture Notes, vol.14,* 1988.

[65] P. Aczel e N. P. Mendler, «A Final Coalgebra Theorem,» in *Category Theory and Computer Science*, London, UK, 1989.

[66] J. J. M. Rutten, «Universal coalgebra: a theory of systems,» *Theoretical computer science,* vol. 249, n. 1, pp. 3-80, 2000.

[67] Y. Venema, «Algebras and co-algebras,» in *Handbook of modal logic*, P. Blackburn, F. J. F. van Benthem e F. Wolter, A cura di, Amsterdam, Elsevier, 2007, pp. 331-426.

[68] J. Van Benthem, «Correspondence theory,» in *Handbook of Philosophical Logic: Vol 2*, D. Gabbay e F. Guenthner, A cura di, Dordrecht, Reidel, 1984, pp. 167-247.

[69] L. S. Moss, «Coalgebraic logic,» *Annals of Pure and Applied Logic,* vol. 96, pp. 277-317, 1999.

[70] L. S. Moss, «Erratum to "Coalgebraic logic". [Ann. Pure Appl. Logic 96 (1999) 277-317],» *Annals of Pure and Applied Logic,* vol. 99, pp. 241-259, 1999.

[71] A. Kurz, «Specifying coalgebras with modal logic,» *Theoretical computer science,* vol. 210, pp. 119-138, 2001.

[72] C. Kupke, A. Kurz e Y. Venema, «Stone coalgebras,» *Theoretical computer science,* vol. 327, pp. 109-134, 2004.

[73] S. K. Thomason, «Reduction of second-order logic to modal logic,» *Mathematical Logic Quarterly,* vol. 21, n. 1, pp. 107-114, 1975.

[74] P. Blackburn and J. van Benthem, "Modal logic: a semantic perspective," in *Handbook of Modal Logic*, P. Blackburn, F. J. F. van Benthem and F. Wolter, Eds., Amsterdam, Elsevier, 2007, pp. 2-85.

[75] S. Abramsky, «A Cook's Tour of the Finitary Non-Well-Founded Sets (original lecture: 1988),» in *Essays in honor of Dov Gabbay. Vol. I*, S. Artemov, H. Barringer, A. d'Avila, L. C. Lamb e J. Woods, A cura di, London, Imperial College Pubblications, 2005, pp. 1-18.

[76] J. Van Benthem, Modal Correspondence Theory. PhD thesis, Dept. of Mathematics, Amsterdam: Amsterdam UP, 1976.

[77] B. Coecke, A cura di, New structures for physics. Lecture Notes in Physics, 813, Berlin-New York: Springer, 2011.

[78] S. Abramsky, «Coalgebras, Chu Spaces, and Representations of Physical Systems,» *J. Phil. Log.,* vol. 42, n. 3, pp. 551-574, 2013.

[79] G. Basti, A. Capolupo e G. Vitiello, «Quantum field theory and coalgebraic logic in theoretical computer science,» *To be submitted,* 2016.

[80] G. Basti, «L'idea di scienza di Maritain fra passato e futuro,» *Aquinas,* vol. 58, n. 1, 2015.

[81] M. Sakellariadou, A. Stabile and G. Vitiello, "Noncommutative spectral geometry, algebra doubling, and the seed of quantization," *Phisical Review, D,* vol. 84, p. 045026, 2011.

[82] G. E. Hinton, S. Osindero e Y.-W. Teh, «A fast learning algorithm for deep belief nets,» *Neural Computation,* vol. 18, n. 7, pp. 1527-1554, 2006.

[83] L. Schröder and D. Pattinson, "Coalgebraic correspondence theory," in *Foundations of Software and Computational Structures. Lecture Notes in Computer Science, vol. 6014*, L. Onge, Ed., Berlin-New York, Springer, 2010, pp. 328-342.

[84] V. Goranko e M. Otto, «Model theory of modal logic,» in *Handbook of Modal Logic*, P. Blackburn, F. J. F. van Benthem e F. Wolter, A cura di, Amsterdam, Elsevier, 2007, pp. 252-331.

[85] A. Bruno, A. Capolupo, S. Kak, G. Raimondo e G. Vitiello, «Geometric phase and gauge theory structure in quantum computing,» *J. of Phys. Conf. Series,* vol. 306, p. 012065, 2011.

[86] "The 2015 Nobel Prize in Physics - Press Release. Nobelprize.org. Nobel Media AB 2014. Web. 13 Dec 2015.

<http://www.nobelprize.org/nobel_prizes/physics/laureates/2015/press.html>," Nobelprize.org, 6 October 2015. [Online]. [Accessed 13 December 2015].

[87] G. Subrata e Al., «Design and construction of a brain-like computer: a new class of frequency-fractal computing using wireless communication in a supramolecular organic, inorganic system,» *Information,* vol. 5, pp. 28-100, 2014.

[88] L. Schröder, D. Pattinson and T. Litak, "A Van Benthem/Rosen theorem for coalgebraic predicate logic," *Journal of Logic and Computation. Online Special Issue, doi:10.1093/logcom/exv043,* vol. 25, 2015.

[89] S. Majid, Quantum groups primer, Cambridge UK: Cambridge UP, 2002.

[90] M. Heller, «Algebraic self duality as the 'ultimate explanation',» *Fundations of science,* vol. 9, pp. 369-385, 2004.

[91] G. Vitiello, «Fractals, coherent states and self-similarity induced noncommutative geometry,» *Physics Leters A,* vol. 376, p. 2527, 2012.